Title:

Eratosthenes: a retrospective current provided

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Abstract

We explain how it was our first participation at Eratosthenes Project (2012). A participation that was full and enriching since within this Project, with 169 participating institutions, we have shared experiences with nearly thirty schools in Latin America and estimated the radius of the Earth 46 times.

We have also endeavoured to contribute our bit especially in improving the calculations and results when observed from cities far removed geographic longitudes. Thus largely correct the mistakes from our position.

We realize that the exchange of experience is always enriching and creates partnerships between people, so with the dissemination of this work we present, we also want this to be a starting point for future editions bridge Eratosthenes and collaboration between Spain and South American people.

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Explanation

Introduction

The Eratosthenes Project 2012 in which we have taken part, was one proposal [1] of the Department of Physics of the Faculty of Exact and Natural Sciences of the University of Buenos Aires (Argentina), Laboratory Pierre Auger, Technological National Regional University Mendoza (Argentina) and Association of Physics of Argentina. The offer was announced and spread at that moment by the NASE Program (Network Astronomy School Education) that was promoted by the Department of Education of the Province of Santa Fe.

To be able to take part we acceded to the web site of The Eratosthenes Project of the National University of Buenos Aires and registered our school there. From his database and by e-mail we could contact with others school centres and arrange some measures. This way we realized our own calculations using a procedure very similar to the one that used Eratosthenes 2300 years ago.

After that, from the University of Buenos Aires, all the information would be analyzed compiled to obtain an average radius of the Earth according to all the measures realized by all the participants in the Project.

Aims of the Project

- To understand the geometric aspects that make the sunbeams affect the Earth in a different way in different latitudes.
- To describe how to determine the solar real midday in the place where one lives.
- To measure the angle that the sunbeams form with the vertical one.
- To describe how the Radius of the Earth measured up for the first time 2300 years ago. To calculate the Earth radius with a very acceptable approximation.
- To form a part of a collective project, from which, with the active dialog of different groups, it is possible to reach a common aim.

A bit of history and theoretical base

Approximately 2300 years ago, the wise Eratosthenes (Cirene, 274 BC - Alexandria, 194 BC) measured, with enough accuracy and for the first time, the size of the Earth. Eratosthenes had news of which, in Siena's city (current Aswan), in the moment of the midday of the summer solstice, the sunbeams were affecting perpendicular towards the soil and were managing to come up to the bottom of a deep well of water. This fact only happened to the midday of this special day from the astronomic point of view. Nevertheless, in the city of Alexandria, to the midday of the summer solstice, the sunbeams were not affecting in a perpendicular way on the land: all the objects were projecting something of shade.

Figure 1. Eratosthenes in Alexandria and Siena (Egypt).

[1] The above mentioned offer was an adjustment from Argentina of the project WYP Eratosthenes Project http: // www.physics2005.org/projects/eratosthenes/TeachersGuide.pdf realized in The United States on the occasion of the International Year of the Physics in 2005.

Eratosthenes thought that, if the Earth was flat, all the objects should project proportional shades in the same instants. Nevertheless, if the Earth was curved (spherical), this difference observed between Alexandria and Siena might have explanation.

Figure 2.
Effect of the curvature.

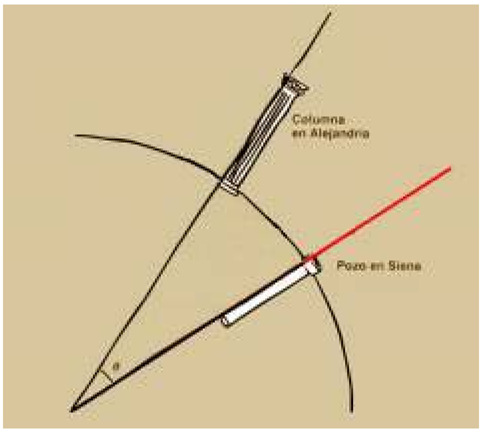

Figure 3. Central angle ***θ***

Eratosthenes was reasoning that the prolongation of the sunbeams in Siena's city (as the low prolongation of all the vertical objects) should come up to the centre of the Earth. Likewise, the low prolongation of all the vertical objects in Alexandria also should come up to the centre of the Earth. Both prolongations should form a central angle ***θ***.

Since the solar beams would come parallel to the Earth, this angle ***θ*** would be easily determinable if we could measure the angle that the sunbeams form with the vertical one in Alexandria (Thales principle).

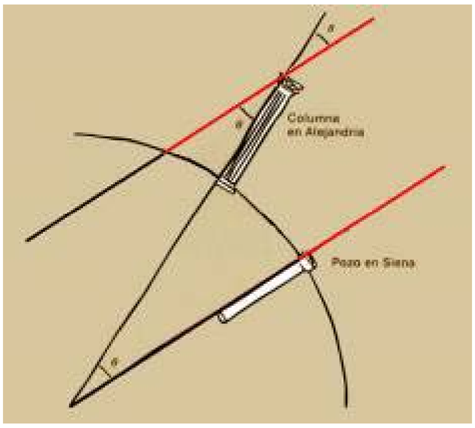

Figure 4. Thales principle.

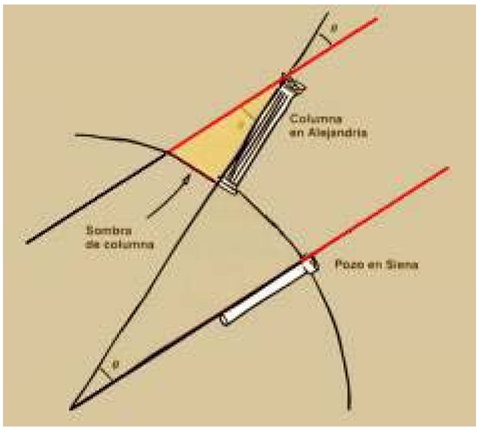

Figure 5. ***θ*** Setting**..**

The angle of inclination of the solar beams in Alexandria can be easily determined by measuring the length of the shade and the height of a vertical column: $\theta = \arctan \dfrac{shadow}{height} = 7{,}2^{\circ}$ (Eratosthenes original measure)

$$7{,}2^{\circ} = 7{,}2^{\circ} \frac{2\pi \cdot rad}{360^{\circ}} = 0{,}126 rad$$

Nevertheless, it is much more interesting to express the angles in radians:

$$7{,}2^{\circ} = 7{,}2^{\circ}\frac{2\pi \cdot rad}{360^{\circ}} = 0{,}126 rad$$

Since a central angle measured in radians comes easily determined as quotient between the length of arch and the radius of the circumference with the one that has been a tracing:

$$\theta = \frac{d}{R}$$

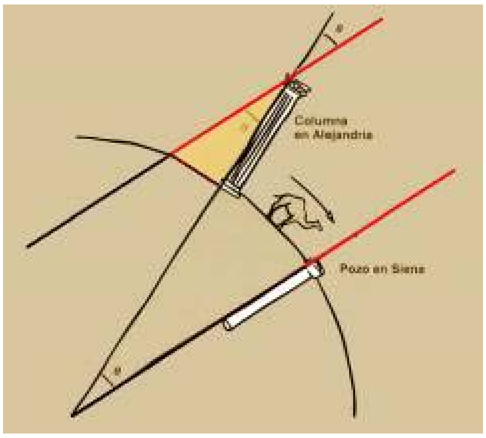

Figure 6.
Distance between Alexandria and Siena

The history tells that Eratosthenes was contracted by a small army in order that he was estimating afoot the distance between Alexandria and Siena.

Distance between Alexandria-Siena (Aswan): 800 Km

And this way:

$$\theta = \frac{d}{R} \Rightarrow R = \frac{d}{\theta} = \frac{800}{0{,}126} = 6366 Km$$

A value very near to the one accepted nowadays (6371 Km).

Method used by our equipment

Firstly we prepare a gnomon as sure as we thought in order to assure uprightness, the horizon and to minimize the mistakes in the measures.

We used a tripod of a small telescope and connected a plastic sheet to the top with an orifice of 1 cm of diameter. This orifice the sunbeams would enter and would be projected on a white horizontal surface. A plumb suspended from the same orifice would provide to us uprightness and the height of the gnomon. The distance from the foot of the plummet up to the luminous point projected in the soil would give us the length of the 'shad' of the gnomon.

A rule, a level, a white paper, a felt-tip pen and a notepad were all the things that we would need.

$$\theta = \arctan\frac{x}{h}$$

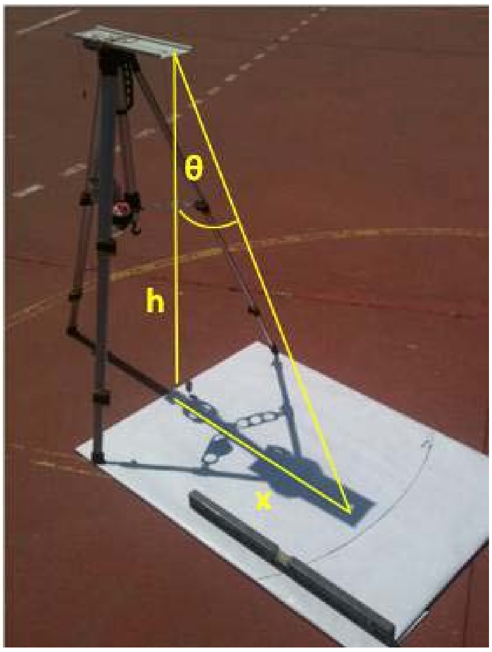

Figure 7. ***θ*** Setting.

Nevertheless, we had to modify a bit the used method because we contacted with cities placed in the south hemisphere.

Each of the observers (we in the North and almost the rest in the South) would determine the inclination of the sunbeams with regard to the vertical observing heights and shades in their respective devices of measurement. They would be the angles $\boldsymbol{\theta_N}$ y $\boldsymbol{\theta_S}$

http://www.wfu.edu/biology/albatross/espanol/gcircle/calcfull.html

Of the other side, each of the observers we would determine the distance from our location to the equator: North distances (Nd) and distances South (Sd). This can be done from Google Earth, Google maps or from the site: http: // www.wfu.edu/biology/albatross/espanol/gcircle/calcfull.html

Providing our latitude (38º N) in the above mentioned application and 0º N for the equator.

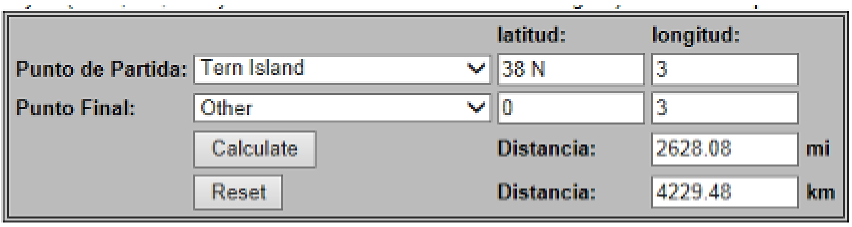

Figure 8. On Line Application for the Equator distance calculation.

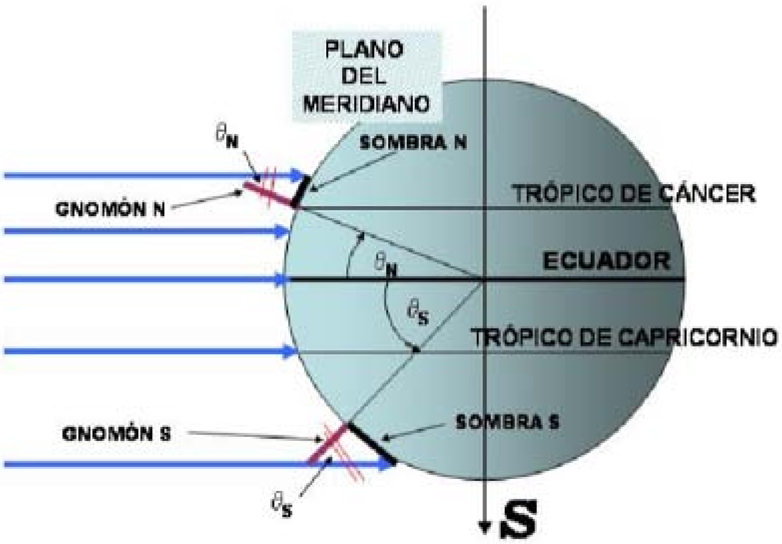

Figure 9. $\boldsymbol{\theta}$ y $\boldsymbol{d}$ as sum of two measurements.

Now, the central angle up to both cities is: $\boldsymbol{\theta} = \boldsymbol{\theta_N} + \boldsymbol{\theta_S}$

And the distance between cities (supposed in the same meridian): $\boldsymbol{d} = \boldsymbol{d_N} + \boldsymbol{d_S}$

And this way, again:

$$\theta = \frac{d}{R} \Rightarrow R = \frac{d}{\theta}$$

The supposition of which both cities are in the same meridian does not drive to very big mistakes since the measurements are realized in both cases in the moment of real midday: when the Sun reaches his maximum altitude on the horizon.

After that, we have designed an improvement in the method with regard to the latter question.

In case both cities are in the same hemisphere, the central angle up to both cities is: $\boldsymbol{\theta = \theta_N - \theta_S}$

And the distance between cities (supposed in the same meridian): $\boldsymbol{d = d_N - d_S}$

But definitively, again:

$$\theta = \frac{d}{R} \Rightarrow R = \frac{d}{\theta}$$

Determination of the real midday

In the simplest theory, the midday should happen at 12:00 h. But it is not true. Of a side, the Greenwich Mean Time of every country gives uniformity to all the clocks without bearing in mind the geographical length of every locality. Of another part, in many countries advances of Greenwich Mean Time are established with regard to the solar time in order to save electric power (an hour or two hours in Spain according to the epochs). And finally, since the Earth does not possess a circular uniform movement, the real midday improve or are slow according to the stations.

día	hora oficial = hora solar + longitud geográfica (Oeste) - ecuación del tiempo						
254	10-sep-12	14h 13m 28s	- 2m 43s	14h 10m 45s		98,3	64,2
255	11-sep-12	14h 13m 28s	- 3m 04s	14h 10m 24s		98,3	65,4
256	12-sep-12	14h 13m 28s	- 3m 25s	14h 10m 03s		98,3	66,3
257	13-sep-12	14h 13m 28s	- 3m 46s	14h 09m 42s		98,3	67,1
258	14-sep-12	14h 13m 28s	- 4m 07s	14h 09m 21s		98,2	67,9
259	15-sep-12	14h 13m 28s	- 4m 29s	14h 08m 58s		98,2	68,8
260	16-sep-12	14h 13m 28s	- 4m 50s	14h 08m 38s		98,2	70,1
261	17-sep-12	14h 13m 28s	- 5m 11s	14h 08m 17s		98,2	70,9
262	18-sep-12	14h 13m 28s	- 5m 33s	14h 07m 55s		nublado	nublado
263	19-sep-12	14h 13m 28s	- 5m 54s	14h 07m 34s		98,2	73,4
264	20-sep-12	14h 13m 28s	- 6m 16s	14h 07m 12s		98,2	74,1
265	21-sep-12	14h 13m 28s	- 6m 37s	14h 06m 51s		98,2	75
						gnomon	sombra
	12h + 2h	Úbeda				(cm)	(cm)
	1º = 4m	3º 22' O					
	1' = 4s	+ 13m 28s					
	15'' = 1s						
			Tabla ecuación del tiempo (Universidad de Alicante)				

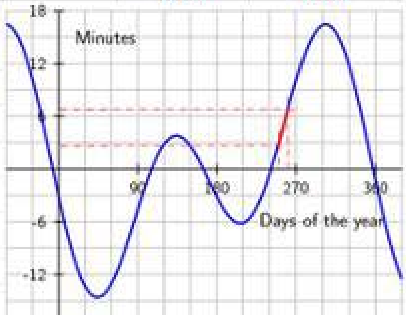

Figure 11. Real noon Calculations and measurements in Ubeda

In the dates when we realize the measurements (September), we had an official advance of 2 h. For 22 were to 3 º 22' to the west of the meridian of Greenwich we should have added up 13min 28s. Of the table of the equation of tiempo2 (University of Alicante[2]) we were remaining some minutes and seconds every day of observation up to determining the Greenwich Mean Time of the real midday.

[2] To see annexe: table equation of the time (Alicante's University)

Timetable activities

The moments of observation were realized in dates before the autumn equinox: between 9 and 21 of September

1.- The 9th of September 2012: Still 8 days were staying to begin our academic course when all our pupils of sciences who are in E.S.O. 4 appeared volunteers to developing the project. This first day we arranged our instruments in the court of our college and determined of experimental form the real midday.

For it we plan an arch of circumference with center in the fall of the plummet. Between 13:00 and 15:00 we were indicating, of ten in ten minutes, the projections of the Sun on the leaf of paper. The most important moments were those in which the Sun was touching the planned curve. The average point of these two, together with the fall of the plummet us determines the direction of the real midday.

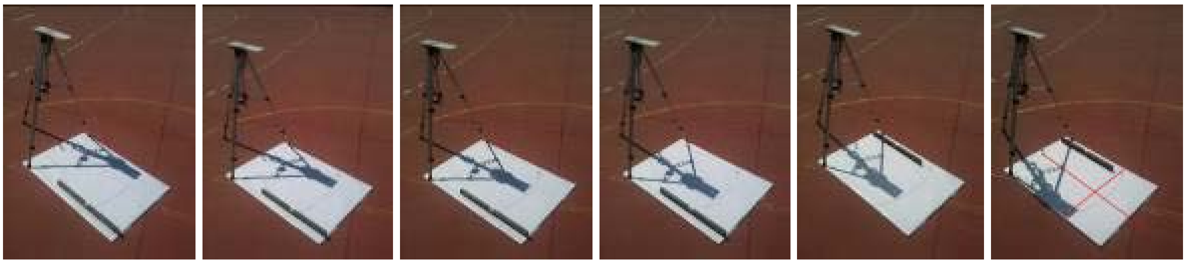

Figure 12. Procedure for calculating the true noon

Though soon we knew that this moment changes day after day and resort, in consequence, to doing calculations of the same one for the following dates (see the previous table in the figure 10).

2.- Every day, from September 10 until September 21, we take measurements and annotate the readings (with the exception of the 18th that one presented very cloudily).

3.- Every day, also from September 10 up to 21, we register our observations in the database of the University of Argentina across his Web. With all of them there would become a statistical general calculation of all the schools participants.

4.- During every September and part of October we were establishing contact with other schools (29 in total), across e-mail, to exchange couples of measures and to realize our individual calculations. We also exchange photography and methods of calculation.

Our students of ESO, so too, could talk to their project partners in other countries of similar ages.

According to the Report Final[3] presented by the University of Argentina:

[3] To see annexe: Formless Final Project Eratóstenes 2012 (University of Argentina)

"In total 167 American schools and two Spanish schools took part. The pupils involved in the activity overcame the number of 12.000. The measurements were realized between 10/09th and 21/09 of September, 2012. The couples of schools that reached to the joint measurement were 110 distributed from agreement to methods of optimization on the basis of the geographical coordinates of all the participants and in the day of measurement.

The values obtained by every couple of schools for the terrestrial radius R changed between 4.000Km and 9.000Km. With the results obtained a histogram was made, and an adjustment was realized by a Gaussian distribution. From the above mentioned adjustment it was obtained as result: R = (6.430 ± 120) Km".

5. - 9 and 10 of April: Presentation of the project in the IV Science Contest in Úbeda.

6. - 11 of May 2013: On the occasion of 18th anniversary of the Park of the sciences of Granada, our pupils exposed his experiences to other centers and to the public in general in the Fair XVI of the Science and Open day. Also they exhibited a short scientist in the Marathon III of scientific documentaries in the classroom inviting to taking part in the new edition 2013 of the Eratosthenes Project in Latin-America.

http://www.parqueciencias.com/parqueciencias/actividades/aniversario2013.html

Our proyect

We have contacted 29 school different centers and have realized 46 couples of measures with his corresponding calculations. We show here a few examples of the calculations realized with some of them:

A	B	C	D	E	F	G	H	I
					RmT:	6371		
		gnomón	sombra	d ecuador	θ (rad)			
11-sep-12	ÚBEDA	98,3	65,4	4227,14	0,587063			
11-sep-12	N Mundo	40	21	2551,85	0,483447			
				6778,99	1,07051	6332,487	-0,60451	%
					RmT:	6371		
		gnomón	sombra	d ecuador	θ (rad)			
11-sep-12	ÚBEDA	98,3	65,4	4227,14	0,587063			
11-sep-12	C S Lucas	100	33,9	2546,29	0,326842			
	México			1680,85	0,260221	6459,314	1,386192	%
				hemos restado (hemisferio Norte los dos)				
					RmT:	6371		
		gnomón	sombra	d ecuador	θ (rad)			
11-sep-12	ÚBEDA	98,3	65,4	4227,14	0,587063			
11-sep-12	Club Astro	52	16	1458,46	0,298499			
	Brasil			5685,6	0,885562	6420,33	0,774296	%
					RmT:	6371		
		gnomón	sombra	d ecuador	θ (rad)			
11-sep-12	ÚBEDA	98,3	65,4	4227,14	0,587063			
11-sep-12	N Iguaçu	40	20	2522,2	0,463648			
	Rio de Janeiro			6749,34	1,050711	6423,596	0,825549	%

Figure 13. Calculations realized with some associate schools

The calculations realized in the Leaf previous Excel follow the guidelines explained in previous points:

1. Of the height of gnomon and his shade we obtain the angle of inclination of the solar beams with regard to the local vertical one (column F) for mediation of: $\theta = \arctan \dfrac{shadow}{gnomon}$ (measured in radians) and do two values add obtained to obtain the central total angle

2. We add the respective distances to the equator to obtain the total distance d (column E).

3. And finally it is calculated: $R = \dfrac{d}{\theta}$ (column G), considering the mistake committed (column H).

Observation: when both cities are in the same hemisphere we reduce angles and distances since already we had mentioned. In many cases we obtained really good results (minor mistakes of 1 %). Besides the reliability of our gnomon, the great distance adds in latitude that separates us from the associate schools. All this does that the mistakes minimize. It is for it for what we would like to spread this experience to other Spanish centres in a near edition. The results of 46 couples of measures can turn in the. **Spreadsheet Excel** to this document

Our particular contribution to the method

Like the associate cities they are very separated in geographical length (let's think here about an average of 60 º) a small mistake of method takes place.

We can observe in the figures 11 and 14 how the minimal length of the shade of every day is increasing every 24 hours. For example: on September 10 it was measuring 64,2 cm, whereas on the 11th it was measuring 65,4 cm. A difference of 1,2 not despicable at all cm. The time of delay between both measures realized in different locations owes to the rotation of the Earth until there is reached the real midday of the second point of observation. If the rotation completes (360 º) he carries an increase of 1,2 cm in the length of our shade, the 60 º (in average) 360 should take an increase of 60 º x 1,2 cm / º = 0,2cm. Since we know our length and those of the associate schools, we can do corrections adding to our shade a small quantity.

That way we simulate that we are on the same meridian and that we realize the measurement in the same instant.

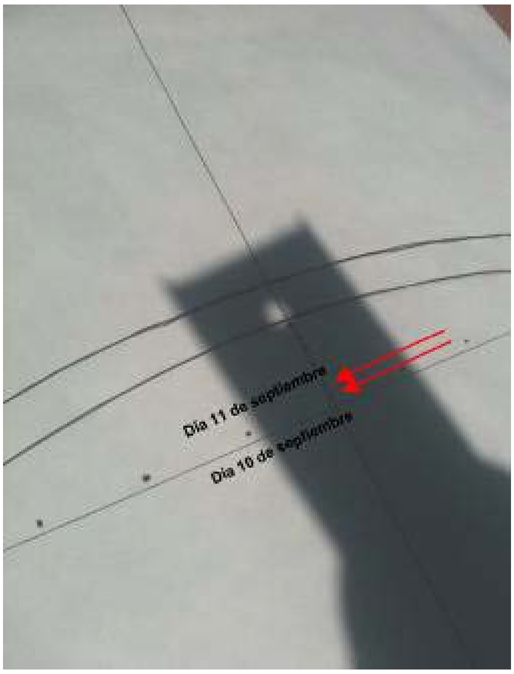

Figure14. Difference between two consecutive days

Let's see some examples of this corrección in the following figures.

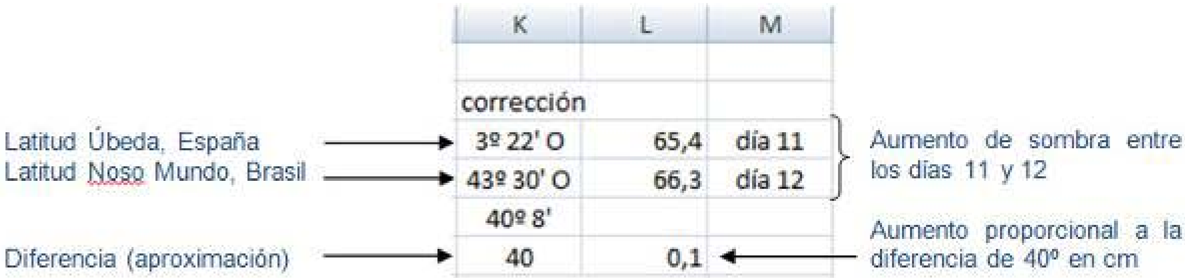

Figure15. Detail of the corrections

A	B	C	D	E	F	G	H	I	J	K	L	M
						RmT:	6371					
		gnomón	sombra	+ correc	d ecuador	θ (rad)				corrección		
11-sep-12	ÚBEDA	98,3	65,4	65,5	4227,14	0,587768				3º 22' O	65,4	día 11
11-sep-12	N Mundo	40	21	21	2551,85	0,483447				43º 30' O	66,3	día 12
	Brasil				6778,99	1,071215	6328,32	-0,66991	%	40º 8'		
										40	0,1	
						RmT:	6371			corrección		
		gnomón	sombra	+ correc	d ecuador	θ (rad)				3º 22' O	65,4	día 11
11-sep-12	ÚBEDA	98,3	65,4	65,7	4227,14	0,589175				109º 51' O	66,3	día 12
11-sep-12	C S Lucas	100	33,9	33,9	2546,29	0,326842				106º 29'		
	México				1680,85	0,262334	6407,299	0,569753	%	106	0,265	
					Aquí restamos (hemisferio Norte los dos)							
						RmT:	6371			corrección		
		gnomón	sombra	+ correc	d ecuador	θ (rad)				3º 22' O	65,4	día 11
11-sep-12	ÚBEDA	98,3	65,4	65,5	4227,14	0,587768				60º 54' O	66,3	día 12
11-sep-12	Club Astro	52	16	16	1465,88	0,298499				57º 32'		
	Brasil				5693,02	0,886267	6423,597	0,825563	%	40,5	0,10125	
						RmT:	6371			corrección		
		gnomón	sombra	+ correc	d ecuador	θ (rad)				3º 22' O	65,4	día 11
11-sep-12	ÚBEDA	98,3	65,4	65,5	4227,14	0,587768				43º 30' O	66,3	día 12
11-sep-12	N Iguaçu	40	20	20	2522,2	0,463648				40º 18'		
	Rio de Janeiro				6749,34	1,051415	6419,29	0,757959	%	40	0,1	

Figure 16.
Some examples with our added corrections

Presence obtained in the media of social communication

They can turn you notify in the matter in:

Diary the ideal one:

http://ubeda.ideal.es/actualidad/2135-salesianos-participo-en-un-proyecto-internacional-de-medicion-conjunta-del-radio-terrestre-ubeda.html (Of Line)

Dpto. of Physics of the University of Buenos Aires:

http://df.uba.ar/7-novedades/6708-proyecto-eratostenes-2012-informe (Of Line)

And in facebook (Project Eratosthenes):

https://www.facebook.com/groups/107311792629457/ (Of Line 1) (Of Line 2)

Our presentation of the project and contribution in the Days IV of Úbeda's Science (April, 2013):

http://www.aaquarks.com/Asociacion_Astronomica_Quarks/Portada/Entradas/2013/2/28_IV_Jornadas_de_la_ciencia_para_tod@s.html

http://www.jaenturismointerior.es/index.php?option=com_content&view=article&id=1972:ubeda-acoge-las-iv-jornadas-de-la-ciencia-para-tods&catid=83:ubeda&Itemid=229

http://diariodigital.ujaen.es/node/34230

http://fundaciondescubre.es/blog/2013/04/10/iv-jornada-de-ciencia-para-tods/

http://salesianosaprendemos.blogspot.com.es/2013/04/iv-jornadas-de-las-ciencias.html

Our contribution and diffusion of the project in the Park of the Sciences of Andalusia - Granada (there was estimated an assistance of 12.000 visitors and the participation of 900 pupils monitors):

http://www.parqueciencias.com/parqueciencias/actividades/aniversario2013.html

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Gallery of images

Rio de Janeiro, Brasil

Palomar, Argentina

Casilda, Santa Fe, Argentina

Quitilipi, Chaco, Argentina

San Cayetano, Buenos Aires, Argentina

Itaperuna, Rio de Janeiro, Brasil

Santa Eufemia, Córdoba, Argentina

Cabo San Lucas, México

C.A.B.A. Argentina

Úbeda, España

Nosso Mundo, Brasil

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References

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http://www.physics2005.org/projects/eratosthenes/TeachersGuide.pdf

2.- Diffusion and access to inscription for the University of Argentina:

http://difusion.df.uba.ar/Erat/erat.htm

Information on the project and didactic material of support: http://www.profisica.cl/proyectos/proyectos-anteriores/proyecto-eratostenes

3.- Basic information in Wikipedia:

http://es.wikipedia.org/wiki/Erat%C3%B3stenes

4.- Table of equation of the time (Alicante university)

http://www.ua.es/personal/viana/Documentos/Astronomia/TablaEcuacionTiempoAnual.pdf

5.- Calculation to the equator distances :

http://www.wfu.edu/biology/albatross/espanol/gcircle/calcfull.html

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