European Association for Astronomy Education

# Catch a Star 2016

## Title:

# First to the moon… then to infinity

**(Transfer orbit of "Hohmann-Chemowsky")**


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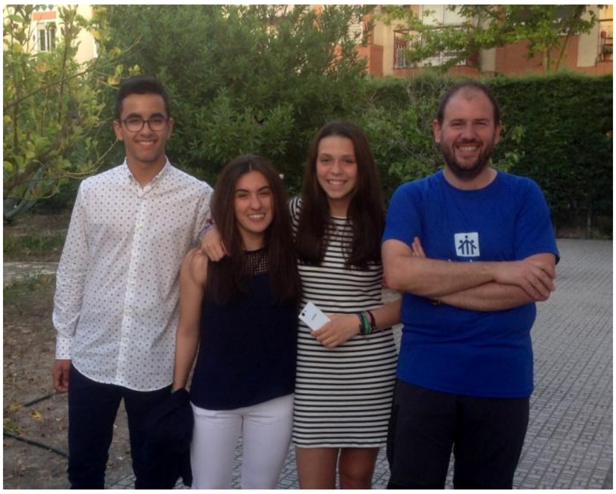


# Abstract

The fundamental idea of our research was to analyze some technical difficulties to be solved when we want planning a trip to take us to another place in the solar system. The viewing of the recent film "The Martian" did made us many questions about the real possibility of a trip.

We found the first clues in some flight parameters of the Apollo missions and met *Hohmann transfer orbits*. If we understand how you can lead a man to the moon (and bring it back to Earth), we can investigate possibilities of how we could find a way to take it anywhere else and bring it back home too.

It is not easy to find technical documentation about it, so this research has been a great challenge for our students. But despite the lack of information, we believe that we have come far: With very humble tools we have managed to get very close, and with good reason, to solve a complex problem that only great specialists - with powerful instrumentation - are trained to solve.

## Introduction

Our students in grades 3 and 4 of ESO only have a few valid calculation tools for the study of Astronomy and Astronautics. Just the Law of Universal Gravitation and some relatively simple equations of motion.

As we will explain, the method of analysis we have devised is simple: by virtue of the position in which our ship is, we can find (moment by moment) the value of the acceleration that it will experience. With this acceleration, we calculate the instantaneous velocity it acquires and the new position to which it will be carried. Again, we will recalculate an updated value of the acceleration (since the position has changed) and, again, new velocities and positions, repeating the process ad infinitum.

With this humble tool based on infinitesimal approximations, we have managed to get close enough, and with good judgment, to the solution of a problem so complex that only great specialists - with powerful instrumentation - are able to solve.

The basic tools we use are:

- The law of Universal Gravitation applied to two and three bodies.
- Knowledge of some physical and orbital parameters of the Earth and the Moon.
- The thrust force of the motors applied at strategic times.
- Own design calculation programs - since there are no others -

Contents developed:

I. Design of work tools and confirmation of their goodness
II. Tests and confirmation of critical flight parameters for a trip to the Moon

We will add a **great** deal of perseverance and patience. Our humble computers have had to make several millions of calculations until we reach the results we will show here.

The computer programs built for this purpose have been made in *Turbo Pascal*. It is an old compiler but allows us to test programs in *exclusive mode* (without more operating system than MS DOS) speeding up all calculation processes. To do it in another way meant to endure desperate times waiting for results.

We document our results using images and the *source code* of the programs (annex).

## The main working tool

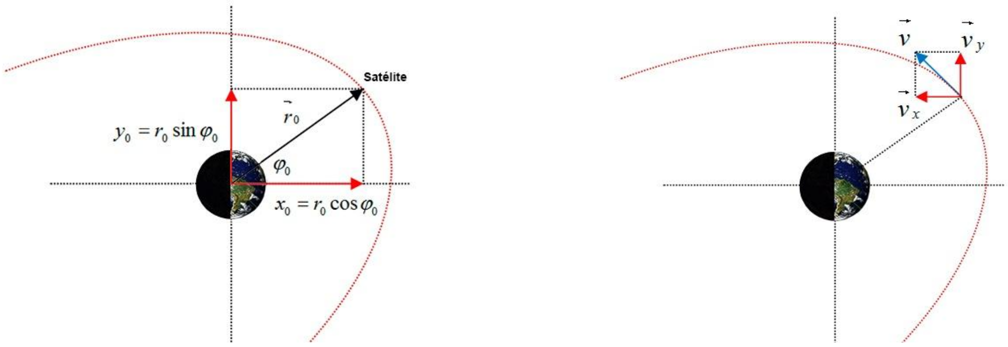
We begin our study by analyzing the orbital motion of any artificial satellite attracted by our Earth (without addressing the problems of the takeoff itself).

And we apply the Law of Gravitation referred to the unit of mass in that satellite that is orbiting around the mass $M_{Earth}$

$$\vec{g} = \frac{\vec{F}}{m_{satélite}} = (-)G\frac{M_{Earth}}{r^2}\vec{u} \qquad G = 6{,}67384 \cdot 10^{-11}\frac{Nm^2}{Kg^2}$$

It is the acceleration (always attractive and central) to which is subjected every Kg of mass that constitutes our artificial satellite or our future spacecraft.

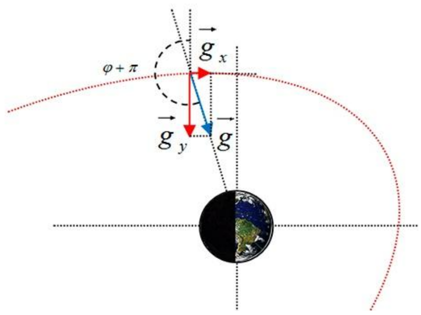
The mathematical treatment begins by establishing any initial position in polar coordinates $\vec{r_0}(r_0, \varphi_0)$ and in Cartesian coordinates $\vec{r_0}(x_0, y_0) = \vec{r_0}(r_0 \cos\varphi_0, r_0 \sin\varphi_0)$. We also start from an initial velocity which, likewise, we express in Cartesian coordinates: $\vec{v_0}(v_{x0}, v_{y0})$




Figures 1, 2 and 3: Decomposition of vectors

When the ship is in $\vec{r_0}(r_0, \varphi_0)$,

we descompose $\vec{g}$ :




$$\begin{cases} \vec{g} = (-)G\dfrac{M}{r_0^2}\vec{u} \\ \vec{g}(g_x, g_y) = \vec{g}(g\cos(\varphi_0 + \pi), g\sin(\varphi_0 + \pi)) \end{cases}$$

(Using $\varphi_0 + \pi$ as argument because $\vec{g}$ is always opposite to $\vec{r_0}$ )

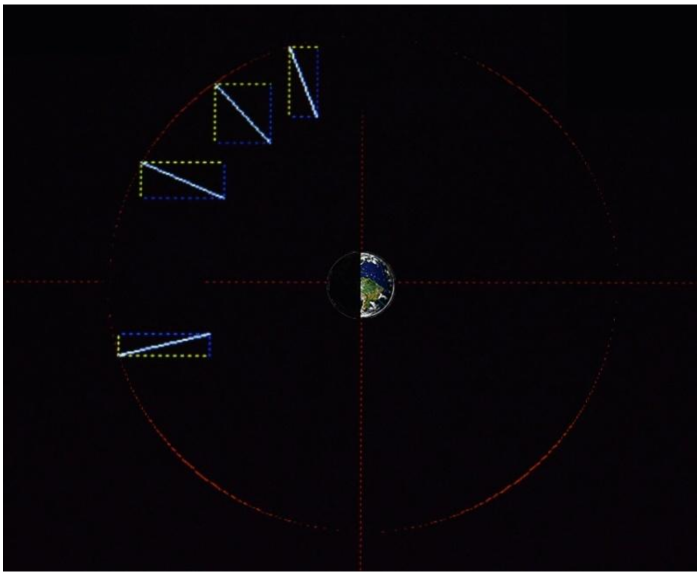


Figure 4: Decomposition of g in real time

Our simple proposal of calculation is to find, moment by moment, a new position of the planet and its new speed through the following equations:

$$\begin{cases} x = x_0 + v_{x0}t + \frac{1}{2} g_x t^2 \\ y = y_0 + v_{y0}t + \frac{1}{2} g_y t^2 \end{cases} \qquad \begin{cases} v_x = v_{x0} + g_x t \\ v_y = v_{y0} + g_y t \end{cases}$$

What are the parametric equations of uniformly accelerated rectilinear movements (we know very well) but, treated infinitesimally, are perfectly valid for our purposes.

The procedure is as can be seen here:

The current coordinates $(x, y)$ that we have found so will serve to locate the planet an instant after the beginning of our treatment and we will make it, in this new instant, the new initial values $(x_0, y_0) \equiv (x, y)$ for the next infinitesimal movement. Also, we will make the speeds $(v_x, v_y)$ Will also be the new initial velocities in the next calculation step $(v_{x0}, v_{y0}) \equiv (v_x, v_y)$.

And we calculate, thus, a new position vector $r_0 = \sqrt{x_0^{\,2} + y_0^{\,2}}$ with its corresponding new argument $\varphi_0 = arctg\left(\frac{y_0}{x_0}\right)$ which will lead us to a new value of acceleration of gravity.

Once again, we perform the Cartesian decomposition of the acceleration:

$$\begin{cases} \vec{g} = (-)G \frac{M}{r_0^{\,2}} \vec{u} \\ \vec{g}(g_x, g_y) = \vec{g}(g\cos(\varphi_0 + \pi), g\sin(\varphi_0 + \pi)) \end{cases}$$

That, just like before, will serve us to find new positions and new speeds (initials also for the next step).

And this process is repeated again and again.

When we consider the attraction of several celestial bodies on our spacecraft, the procedure is very similar to that explained here, except that the acceleration that will act on it will be the vectorial sum of the accelerations experienced. In particular:

$$, \quad \vec{g}_{Earth} = (-)G\frac{M_{Earth}}{r_{0E}^2}\vec{u}_E \,, \qquad \vec{g}_{Moon} = (-)G\frac{M_M}{r_{0M}^2}\vec{u}_M$$

$$\begin{cases} \vec{g}_{Earth+Moon}(g_x, g_y) \\ g_x = g_E \cos(\varphi_{0E} + \pi) + g_M \cos(\varphi_{0M} + \pi) \\ g_y = g_E \sin(\varphi_{0E} + \pi) + g_M \sin(\varphi_{0M} + \pi) \end{cases}$$

(With position vectors referred to each attraction center and arguments referred to them as well)

Obviously, the calculation processes are very delicate and tremendously repetitive so we made use of the power of a small computer, implementing computer programs dedicated to multiple tasks that did all the operations for us.

## The trip

As we have already commented, it is not easy to find technical documentation about it that is also understandable at short ages, so this research work we present has been a great challenge for us. Even so, we try the best we can address.

**We present you our own flight plan.**

The first step to placing a man on the Moon is to place a spacecraft in a circular orbit low around the Earth. Specifically, to 300 km of height and with a speed of 7730 m / s

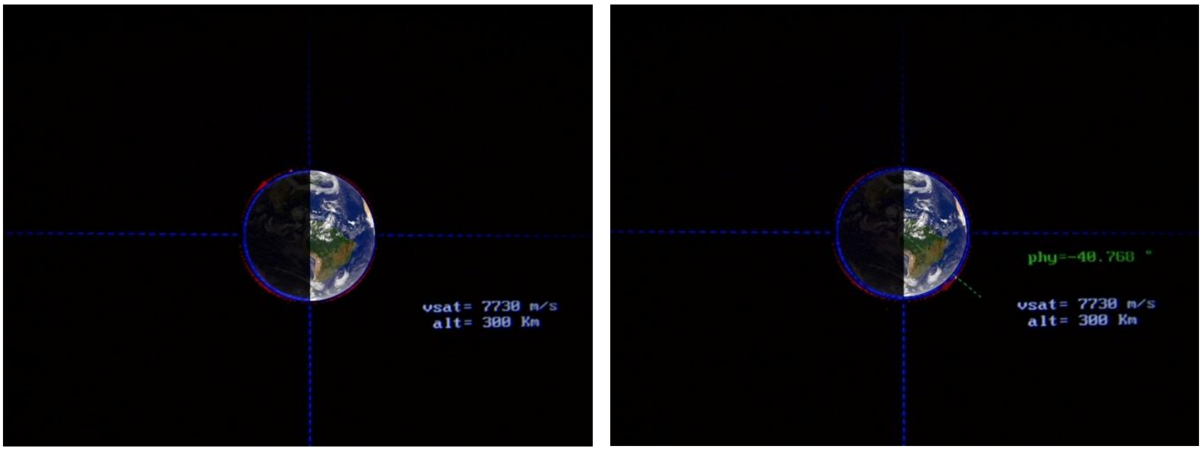
Our program of calculation and movement simulator allows us to vary the speed with which our ship would move and, in fact, we have done many tests of it until finding a circular orbit. Our value of 7730 m / s is an empirical value and found with patience and expectation.

However, it must be said that this orbital velocity can be calculated theoretically by:

$$v_{orb} = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6{,}67384x10^{-11} \cdot 5{,}9736x10^{24}}{6{,}672x10^{6}}} = 7729{,}972 \text{ m/s}$$

Where the values of *G* and *M* have been taken from verified sources of information and the value of

$$r = R_{Earth} + height = 6{,}372x10^6 + 0{,}300x10^6 = 6{,}672x10^6 \text{ m}$$

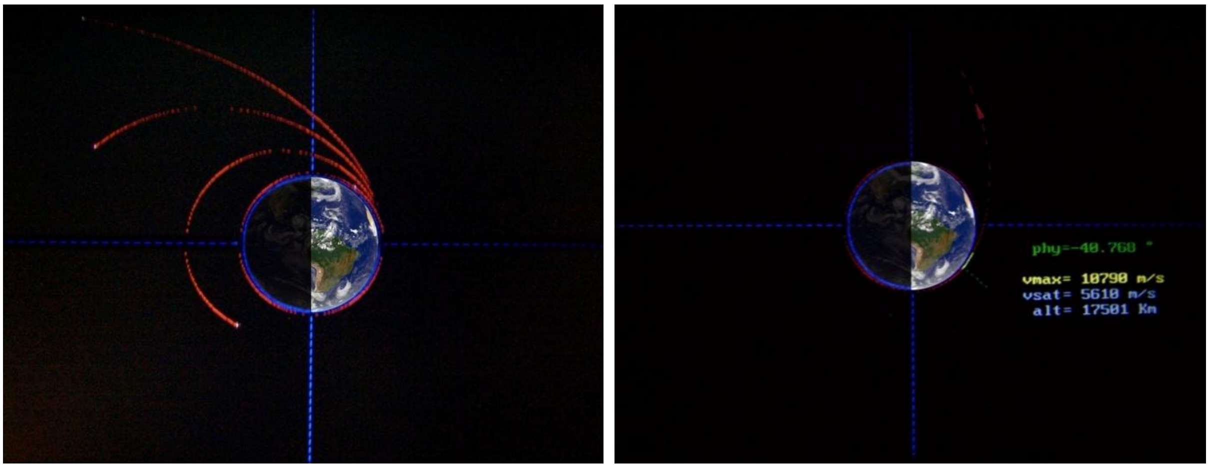



Figures 5 and 6: Preparation orbit and launch window

At a very specific moment - which engineers call the launch window - we must light the ship's engines for 210 s (three and a half minutes) until we reach a maximum speed of 10790 m / s.

| | | | $t_{acel}$: 210 s | | |
|---|---|---|---|---|---|
| $h_0$ | $V_0$ | $\Phi_0$ | $a_x$ | $a_y$ | $V_{final}$ |
| 300 Km | 7730 m/s | -40.768º | 14.845 cos(Φ+π/2) | 14.845sen(Φ+π/2) | **10790 (m/s)** |

Note that we perform a tangential acceleration of 14,845 m/s$^2$ during 210 s.




Figures 7 and 8: Choice of final momentum for the Earth-Moon transfer

But why even up to that speed precisely? Well, to arrive - with a minimum amount of energy - right up to the distance to which the Moon orbits (a little more actually).

With less speed we will be short and we would not arrive; With more speed we would save time but, in the end, we would have to slow down not to surpass the Moon and not to lose the objective. And this quick maneuver would be an unnecessary waste of energy that is not convenient.

In our sources of information we find only **some clues** about the flight parameters of these historical trips. These parameters can be calculated theoretically as well but they exceed - and by far - the possibilities of real understanding of our students.

Again, and only from a thrust of three and a half minutes and the distance Earth-Moon, we did many tests until getting the proper acceleration and propitious moment of ignition of motors.

After many failed attempts, little by little we were approaching to results more and more valid. A patient job - with emotion, it must be said - and great fun.

We also observe in the previous figures that we begin our launching moment just when $\phi_0$ = -40,768º

This moment is known as opportunity or *launch window* and is the one that will take us to the direction of the Moon when, after several days of travel, we approach her (and she, in turn, to us due to her own displacement In that time of transit).

Actually, this window has a small margin of variation but it is not too large.

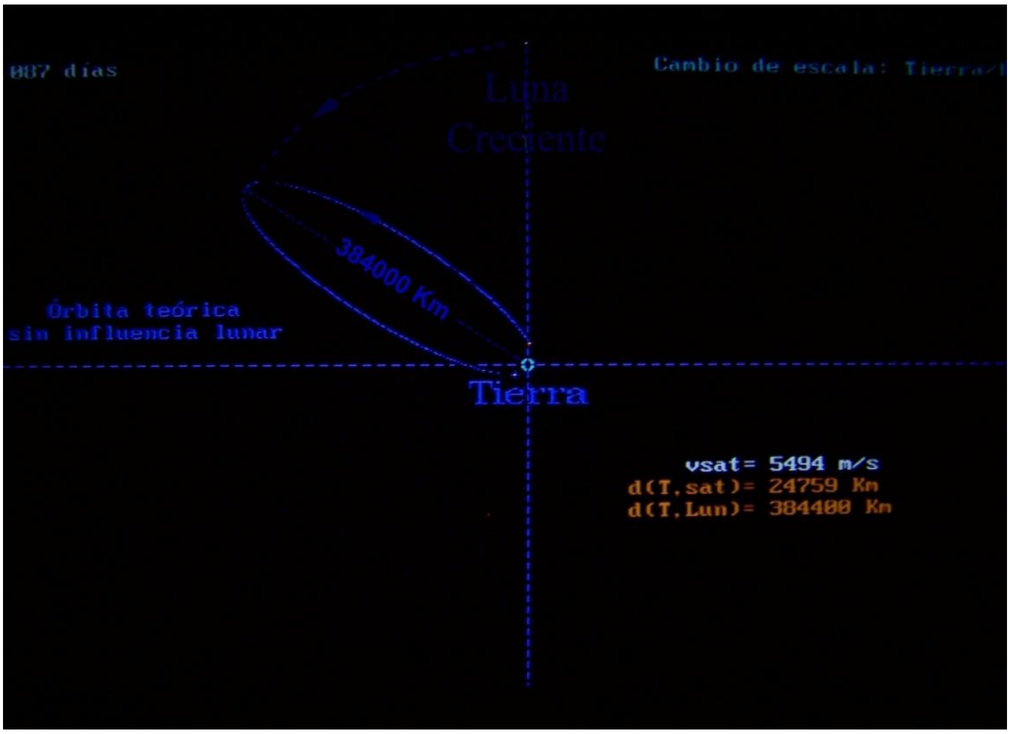



Figure 9: Theoretical orbit without taking into account the lunar influence

Our trip should describe an elliptical orbit as represented in the previous figure with *periastro* to 300 km of the surface of the Earth (6671 km from the center of the Earth) and with an *apoastro* located at the distance of the Moon. A little more, in fact, because we want to surpass the Moon a little and maneuver from its hidden face (as we can see).

The trip will take three days (actually, almost four). Time that, although we have simply visualized it with our program, can also calculate:

Constant of Universal Gravitation $G = 6{,}67384 \cdot 10^{-11}$ $Nm^2/Kg^2$

Mass of Earth $M = 5{,}9736 \cdot 10^{24}$ Kg

Semiaxe major of the orbit that we will describe $a = \dfrac{384400000 + 6671000}{2} = 19535500$ m

Orbital period $T = \frac{2\pi}{\sqrt{GM}} a^{3/2} = \frac{2x3{,}14159}{\sqrt{6{,}67384 \cdot 10^{-11} x5{,}9736 \cdot 10^{24}}} 19535500^{3/2} = 27171$ s

That is 7,548 days in realizing a complete orbit.

But our trip will only take half: 3,774 días.

And let's continue with our journey:

In the vicinity of the apoastro, we see that our spacecraft departs from the theoretical orbit drawn by turning to the right. This is because the Moon is already very close to us and begins to exert a remarkable attraction.

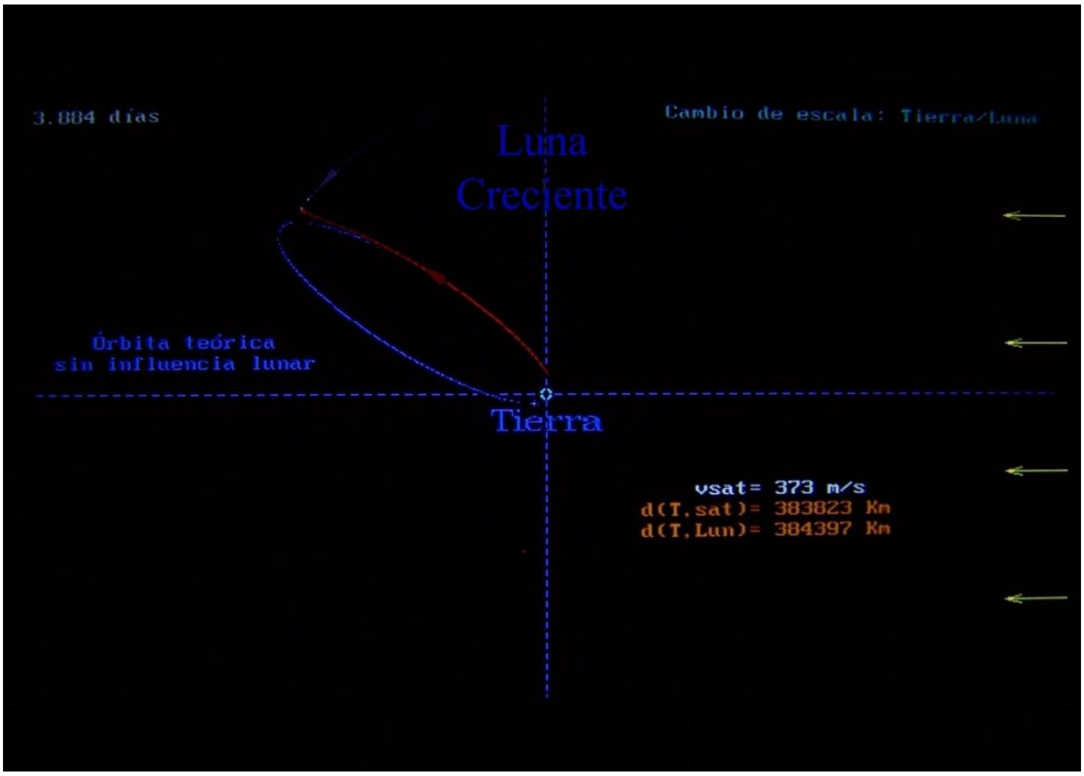



Figure 10: Deviation caused by the Moon. Hohmann orbit

At the point of maximum distance from Earth, our spacecraft has very little speed and is moving to the right attracted by the Moon.

If we did not do anything, the Moon would pass in front of the spacecraft and, without much more consequences, our space ship would fall again towards Earth.

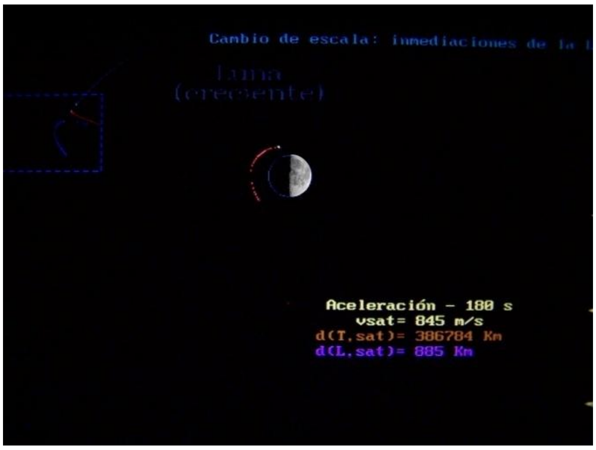



Figure 11: Lunar atraction

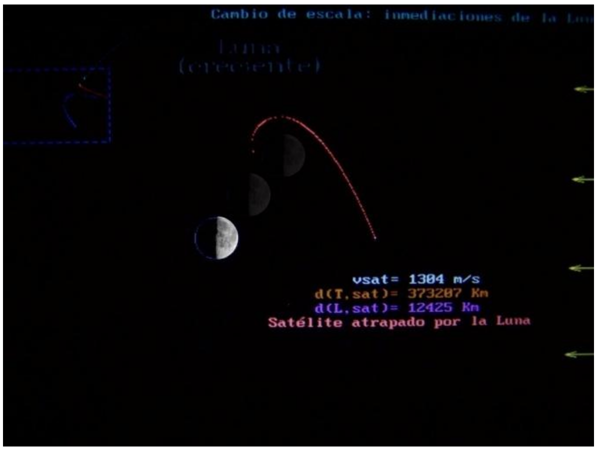
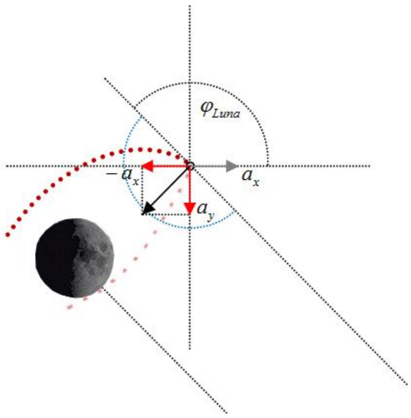



Figure 12: Theoretical orbit without braking maneuver

But in a very precise moment - and with a very precise orientation too - we turn on the engines for 180 s (exactly three minutes). This maneuver makes it possible for us to follow the Moon's progress with a little more speed than it, and that we pass almost touching its surface (34 km in height).

There is no problem in this because our Moon has no atmosphere.


Figures 13 and 14: Impulse of approach and trapped by the Moon

Once reached that minimum of height, the ship moves away from our satellite until reaching a maximum height of 1063 km. Our ship will return again to descend being trapped by the gravitational field of the Moon in an elliptical orbit.

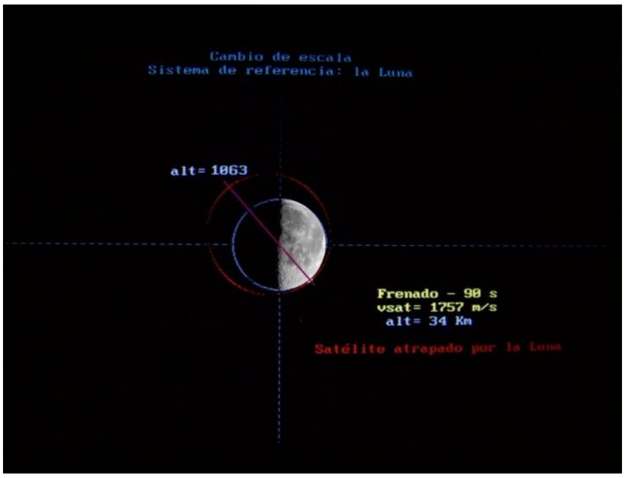



Figure 15: Eliptic orbit

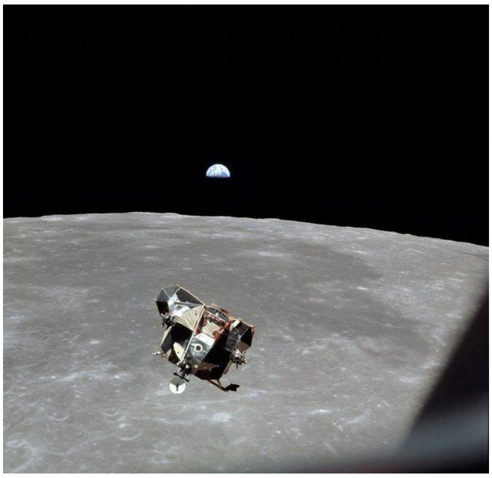
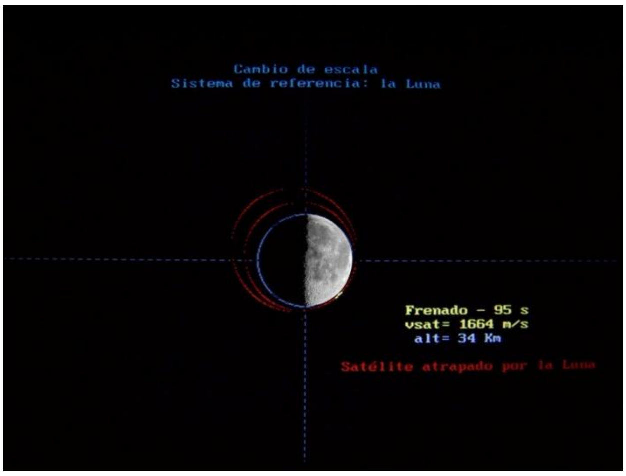
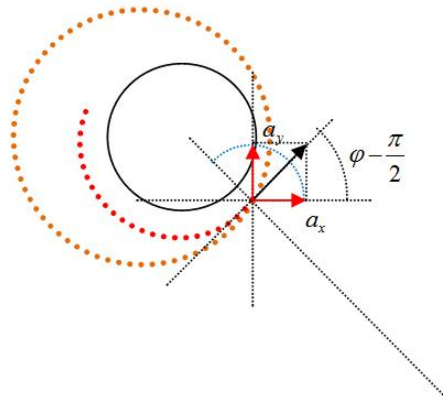


Figure 16: Eagle in orbit - Apollo XI

We have little to achieve our goal.

At the point of minimum height we make a first braking that will last 90 s. This allows the orbit to become a little more circular.




Figures 17 y 18: Two brakes to low circular orbit

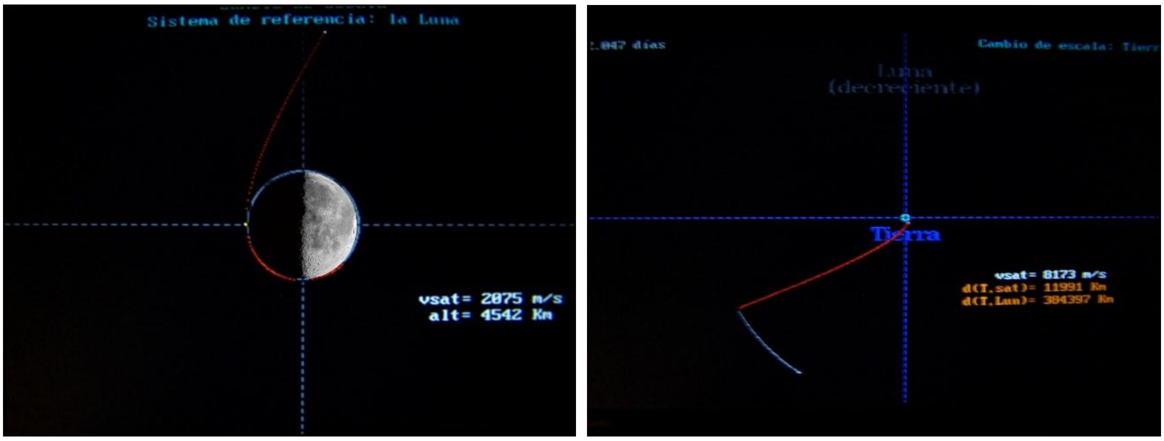
And something later, we made a second braking of 95 s to obtain a practically circular orbit. It is from this orbit from where we can descend to the Moon and, later, begin the return to Earth.

## The return at home

The stay at La Luna - if we stick to the historical trips made in the 70's - would last a few days. Although, really, there is no limit to it; It only depends on the amount of consumable resources brought in there.

For the return we must access from the surface of the Moon to the ship that we leave in orbit and, from it, initiate a delicate maneuver of escape from the gravitational field of the Moon.

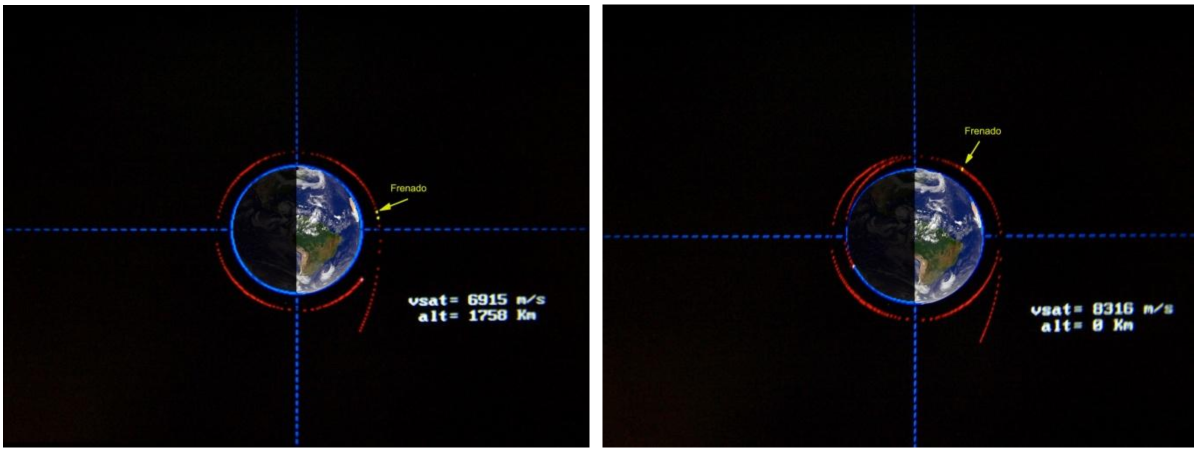
We do this by making a tangential launch so that, seen from Earth's reference system, we rest speed on our spacecraft and let the Moon escape so it can continue on its way. After this maneuver, we began to fall towards the Earth.




Figures 19 y 20: Escape from the Moon and transfer back to Earth

The launch window for the return we have considered a few days after the Full Moon. When the Moon is 7π / 6. The impulse of our engines was 5.8 m / s2 carried out for 210 seconds.

All these parameters, although theoretically calculable, for us - we repeat - are the fruit of many tests until we find the path that takes us very close to the Earth.




Figures 21 y 22: Capture by the Earth and descent to the surface

A first braking maneuver stabilizes our trajectory within a circular safety orbit (210 seconds with tangential acceleration of 14,845 m / s2). And a much lower final braking - with proper orientation - will make us descend through the atmosphere in that only form we know how to do.

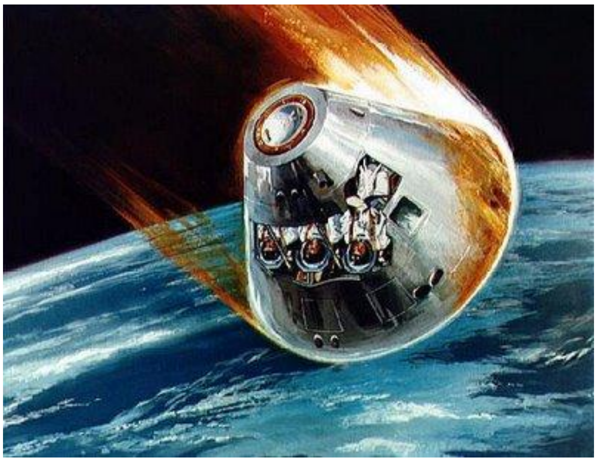


Figure 23: Reentry in the atmosphere

## Results and conclusions

Actually, the study that we have dealt with is very complex and the work done until obtaining critical parameters has been very great. It is necessary to point out, once again, that it has taken us a long time to reach values that (we believe) are correct.

The secondary title of this research paper - "Transfer orbir of Hohmann-Chemowsky" - refers to our own ideas, corrections and contributions in all stages of the mission.

The method of calculation that we have employed of "approximations to uniformly accelerated rectilinear movements" is perfectly valid when the time used in each of the steps tends to zero. This way of doing has led us to have to expect very large computational times most of the time.

But the truth is that we are not afraid of it: we know very well that gravitational studies with three or more bodies have no analytical solutions and must necessarily be performed numerically and step by step. It is not a whim that there are professional work teams dedicated to the calculation in each one of the real missions and that the powerful computers with the space agencies are needed for it.

However, our results - we know very well - are more qualitative than quantitative, and are only intended to verify our capacity for analysis in this complicated research. With all of this, we believe that our work shows a fairly acceptable and credible reality.

# Referencies


Some useful calculations: http://www.sc.ehu.es/sbweb/fisica/celeste/kepler3/kepler3.html

Gravitational assistance: https://es.wikipedia.org/wiki/Asistencia_gravitatoria

Constant G: https://es.wikipedia.org/wiki/Constante_de_gravitaci%C3%B3n_universal

Data of the Moon: https://es.wikipedia.org/wiki/Luna

Data of the Earth: https://es.wikipedia.org/wiki/Tierra

Transfer orbit: https://es.wikipedia.org/wiki/%C3%93rbita_de_transferencia_de_Hohmann

Velocidad orbital: https://es.wikipedia.org/wiki/Velocidad_orbital


Annexed programs (Source code, files PDF), 2016
LUNA.PAS
RETURN.PAS
ORBITS.PAS